#  Trigonometric Principles Instructional

#  Handout - Virtual Roller Coaster Design

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**Objective:** To apply trigonometric principles in the design and analysis of roller coaster tracks.

## Understanding Sine, Cosine, and Tangent

### Definitions

In a right triangle, sin(θ) = opposite/hypotenuse.
In a right triangle, cos(θ) = adjacent/hypotenuse.
In a right triangle, tan(θ) = opposite/adjacent.

### Applications

Calculate angles and lengths in track designs, especially for turns and slopes.

## The Pythagorean Theorem and its Applications

### Formula:

###  $a^{2}+b^{2}=c^{2}$

### Applications

Determine the length of a track segment when two sides of a triangle are known.
Essential in calculating the height and base lengths of inclines and declines.

## Calculating Slopes and Angles of Incline in Roller Coaster Tracks

### Formulas and Techniques

Use trigonometric ratios to calculate the angle of inclination or decline.
Apply tangent for finding angles: tan(θ) = rise/run.

## Applications in Non-Right Triangles in Roller Coaster Design

### Law of Sines

 $\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$

### Law of Cosines

 $c^{2}=a^{2}+b^{2}+2ab\cos(C)$

### Applications

Solve for unknown sides and angles in tracks where no angle is 90 degrees.
Useful in designing complex curves and intersections.

## Examples in Calculating Track Lengths and Angles

**Example 1:** Calculating the height of a hill using the Pythagorean Theorem.
**Example 2:** Determining the angle of descent using tangent.
**Example 3:** Using the Law of Sines to find the length of a curved track section.

## Exercises and Examples in Applying Trigonometry to Roller Coaster Design

### Exercise 1

As part of the design for a new roller coaster, you are tasked with calculating the angle of inclination for one of the slopes. This particular slope will be a thrilling feature where the coaster ascends before a major drop. You are given the following information:

* The vertical height (rise) of the slope is 80 feet.
* The horizontal length (run) of the slope is 150 feet.

Using this information, calculate the angle of inclination for the slope.

### Exercise 2

In the design of a new roller coaster, you are tasked with creating a non-regular, triangular-shaped loop. Unlike a standard circular loop, this loop will have three straight segments forming a triangle. You need to calculate the length of the third side of the loop, knowing the lengths of two sides and the angle between them.

You are given the following information:

* Length of the triangular loop's first side (side a): 100 feet.
* Length of the triangular loop's second side (side b): 150 feet.
* The angle (C) between sides a and b: 40 degrees.

Using the Law of Cosines, calculate the length of the loop's third side (side c).

### Exercise 3

You are designing a roller coaster section and need to calculate the height of a particular drop. For this section of the track, you know the length of the slope (hypotenuse) and the angle of descent. Using the sine ratio, determine this coaster section's vertical height (drop).

Given data:

* Length of the slope (hypotenuse): 250 feet.
* Angle of descent: 30 degrees.

Calculate the vertical height of the coaster drop.

## Solution to Exercises and Examples

### Solution 1

We will use the tangent trigonometric ratio to find the angle of inclination. The tangent of an angle in a right triangle is the ratio of the opposite side (rise) to the adjacent side (run).

The formula for tangent is:

 $\tan(θ=\frac{Opposite Side}{Adjacent Side})$

Where $θ $is the angle of inclination we want to find.

In this scenario:

* Opposite Side (rise) = 80 feet
* Adjacent Side (run) = 150 feet

Thus, the formula becomes:

 $\tan(θ=\frac{80}{150})$

 Now, to find the angle, $θ,$ we take the arctangent (inverse tangent) of both sides:

 $θ=tan^{-1}\frac{80}{150}$

Using a calculator, calculate $tan^{-1}\frac{80}{150}$ to find the angle in degrees.

### Solution 2:

The Law of Cosines is used to find a side of a triangle when you know the lengths of two other sides and the angle between them. The formula for the Law of Cosines is:

 $c^{2}=a^{2}+b^{2}+2ab\cos(C)$

In this problem:

* $a=100 ft.$
* $b=150 ft.$
* $C=40°$

Plug these values into the formula:

 $c^{2}=100^{2}+150^{2}+\left(2\right)\left(100\right)\left(150\right)\cos(40°)$

Now, calculate the value of $c^{2}$ and then find $c$ by taking the square root:

 $c=\sqrt{100^{2}+150^{2}+(2)(100)(150) cos⁡〖40°〗}$

### Solution 3

To calculate the height of the drop, we will use the sine trigonometric ratio. The sine of an angle in a right triangle is the ratio of the length of the side opposite the angle (the drop height in this case) to the length of the hypotenuse (the slope).

The formula for sine is:
 $\sin(θ=\frac{Opposite Side}{Hypotenuse})$

Where $θ$ is the angle of descent. Rearranging the formula to solve for the opposite side (height of the drop), we get:

 $Opposite Side=\sin(θ×Hypotenuse)$

In this scenario:

θ (angle of descent) = 30 degrees

Hypotenuse (length of the slope) = 250 feet

Thus, the height of the drop can be calculated as:

Height of Drop = $\sin(30°×250)$

Using a calculator, calculate $\sin(30°×250)$ to find the height of the drop in feet.