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A NEW CLASS OF FIXED-POINT METHODS FOR SOLVING ABSOLUTE VALUE EQUATIONS*

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Abstract. This paper develops a new class of fixed-point iterative methods for solving the absolute value equation (AVE) of the form $Ax-|x|=b$. The proposed approach is based on two distinct splittings of the matrix A, which are used to construct a block system of nonlinear equations. Sufficient conditions are provided for (unique) solvability of the underlying block system, ensuring that any solution to this system also yields a solution to the given AVE. A Kellogg-type class of iterative methods is further developed for solving the considered AVE, and the convergence properties of the proposed approach are established. Additionally, numerical experiments are disclosed to illustrate superiority of the proposed class of iterative methods over two recently proposed methods in the literature.

Key words. Iterative methods, Absolute value equation, Solvability, Splitting, Convergence.

AMS subject classifications. 65F10, 90C05.

1. Introduction.

Consider the system of absolute value equations (AVEs) given by

$$Ax - |x| = b, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^n$ are given, $x \in \mathbb{R}^n$ is unknown, and $|x|$ denotes the component-wise absolute value of x. AVEs arise in various fields such as analyzing linear programming problem, bimatrix games, linear complementarity problem, and several other applications in scientific computing and engineering, see [1, 4, 5, 13, 14, 16, 17, 22]. It is known that determining the existence of a solution to the AVE is NP-hard [12], and if it is solvable, it can have either a unique solution or multiple solutions, see [8] and the references therein for further details.

It is worth noting that the sufficient condition $\nu := \|A^{-1}\| < 1$[1] ensures the existence of the unique solution for the AVE (1.1), see [13] for more details. Under the assumption $\nu < 1$, several iterative schemes recently proposed by rewriting the AVE as a block system of equations. For instance, the SOR-like method for solving (1.1) is developed in [6, 11]. Shams and Beik [19] designed the following fixed-point iterative method:

$$\begin{cases} x^{(k+1)} = (A+Q)^{-1}(Qy^{(k)} + b), \\ y^{(k+1)} = (1-\tau)(Q^{-1}|x^{(k)}| + x^{(k)}) + \tau(Q^{-1}|x^{(k+1)}| + x^{(k+1)}), \end{cases} \tag{1.2}$$

where Q is an auxiliary nonsingular matrix, the initial guess $x^{(0)}$ is given and $y^{(0)} = Q^{-1}|x^{(0)}| + x^{(0)}$. Here, $\tau > 0$ is a prescribed parameter.

More recently, Chen et al. [3] proposed the exact and inexact Douglas–Rachford splitting (DRS) methods for solving large-scale sparse absolute value equations. In particular, the exact DRS method for solving (1.1) can be described as follow:

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[1]The norm $\|.\|$ refers to the matrix (vector) 2-norm.

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$$x^{(k+1)} = \left(1 - \frac{1}{2}\tau\right) x^{(k)} + \frac{1}{2}\tau A^{-1}(|x^{(k)}| + b), \tag{1.3}$$

where the initial guess $x^{(0)}$ and the parameter $\tau > 0$ are given. Convergence of iterative method (1.3) was proved for $\tau \in (0, 2)$. While this interval provides a sufficient condition for convergence, our numerical experiments illustrate that the optimal value of τ, leading to fastest convergence speed of (1.3), may lie outside interval $(0, 2)$; see Example 3.1 for details. In [3, Section 5], it has been numerically seen that the above iteration method outperforms the SOR-like method [11] for solving (1.1).

In this paper, we first consider an alternative block system of nonlinear equations in our theoretical discussions. Sufficient conditions are presented under which the mentioned block system has a (unique) solution. It is seen that if $(x^*; y^*; z^*)$[2] is a solution of block system, then $x^* + y^*$ satisfies (1.1). Using a particular splitting for the coefficient matrix of block system, we develop a class of iterative methods to solve (1.1). It is worth mentioning that the block system leads to a three-step iterative scheme, and there is no need to form it explicitly in practice. Numerical results are reported to show that the proposed method can surpass the recently examined methods in [3, 19]. The developed approach is inspired by the recently proposed Kellogg version of HSS iterative method [20] for solving linear system of equations of the form $Ax = b$, whose convergence properties have been analyzed with respect to arbitrary splittings in [2].

The remainder of this paper is organized as follow: Before ending this section, we present some utilized definitions and notations throughout the paper. In Section 2, we propose a new class of iterative schemes for solving (1.1) and establish sufficient conditions for their convergence. Section 3 is devoted to reporting some numerical comparisons to show the effectiveness of new method over the recently proposed methods in [3, 19]. Finally, the paper is ended with brief concluding remarks in Section 4.

Given a square matrix A, the decomposition $A = M - N$ is called *splitting* if M is nonsingular. For a given vector $x \in \mathbb{R}^n$, the notation "sign(x)" represents a vector whose components take values $1, 0$, and -1 depending on whether the corresponding component of x is positive, zero, or negative, respectively. The matrix $D(x) := \text{diag}(\text{sign}(x))$ indicates the diagonal matrix formed from sign(x). The transpose of x is represented by x^T. Given an arbitrary square matrix S, the notation $\rho(S)$ denotes its spectral radius.

2. Main results. In the following, we present a class of iterative methods for solving (1.1) and study its convergence properties. As a consequence of the established results, we derive sufficient conditions for the solvability of (1.1).

We begin by considering the following equivalent form of the AVE problem in (1.1):

$$Ax - D(x)x = b. \tag{2.4}$$

Invoking two splittings $A = M_1 - N_1 = M_2 - N_2$, we consider the following alternative system of equations

$$M_1\bar{x} - N_2\bar{y} - \gamma\bar{z} = b, \tag{2.5a}$$

$$-N_1\bar{x} + M_2\bar{y} = 0, \tag{2.5b}$$

$$D(\bar{x} + \bar{y})(\bar{x} + \bar{y}) - \gamma\bar{z} = 0, \tag{2.5c}$$

[2]For given vectors x, y, and z of dimensions n, the MATLAB notation $(x; y; z)$ refers to a column vector of dimension $3n$.

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where $\gamma > 0$ is a prescribed parameter. The following proposition shows that every solution of (2.5) corresponds to a solution of (1.1).

PROPOSITION 2.1. *If $(\bar{x}^*; \bar{y}^*; \bar{z}^*)$ be a solution of (2.5), then $\bar{x}^* + \bar{y}^*$ satisfies (1.1).*

Proof. Let $(\bar{x}^*; \bar{y}^*; \bar{z}^*)$ be a solution of (2.5). Adding (2.5a) to (2.5b) yields

$$\begin{aligned} b &= (M_1 - N_1)\bar{x}^* + (M_2 - N_2)\bar{y}^* - \gamma\bar{z} \\ &= A\bar{x}^* + A\bar{y}^* - \gamma\bar{z}^* \\ &= A(\bar{x}^* + \bar{y}^*) - \gamma\bar{z}^*. \end{aligned} \tag{2.6}$$

We comment that Eq. (2.5c) implies $\bar{z} = \frac{1}{\gamma}D(\bar{x}^* + \bar{y}^*)(\bar{x}^* + \bar{y}^*) = \frac{1}{\gamma}|\bar{x}^* + \bar{y}^*|$. By Eq. (2.6), one can conclude that if $(\bar{x}^*; \bar{y}^*; \bar{z}^*)$ satisfies (2.5), then $\bar{x}^* + \bar{y}^*$ is a solution of AVE. ∎

Here, a natural question to consider is whether the nonlinear system (2.5) has a solution. First, in the following theorem, we provide sufficient conditions for the system (2.5) to have a unique solution within a specific compact set.

THEOREM 2.2. *Consider the following set of vectors of size $3n$,*

$$\mathbb{S} = \left\{ (x_1; x_2; x_3) \in \mathbb{R}^{3n} \;\middle|\; \|x_1\| + \|x_2\| + \|x_3\| \leq \frac{\delta\,\|b\|}{1-\kappa} \quad \text{for } x_1, x_2, x_3 \in \mathbb{R}^n \right\}, \tag{2.7}$$

where

$$\kappa := \max\{\alpha + \gamma^{-1}, \beta + \gamma^{-1}, \delta\gamma\}, \tag{2.8}$$

with

$$\alpha := \|M_1^{-1}N_2\|, \quad \beta := \|M_2^{-1}N_1\|, \quad \text{and} \quad \delta := \|M_1^{-1}\|. \tag{2.9}$$

The system of equations (2.5) is uniquely solvable on $\mathbb{S}$, if the following conditions satisfy

$$\max\{\alpha, \beta\} + \gamma^{-1} < 1, \tag{2.10}$$

and

$$0 < \gamma < \frac{1}{\delta}. \tag{2.11}$$

Proof. Note that the assumptions (2.10) and (2.11) ensure that $\kappa < 1$. Consider the mapping $\mathcal{T}$ on $\mathbb{R}^{3n}$ defined as follows:

$$\mathcal{T}(\mathbf{x}) = \left(M_1^{-1}N_2x_2 + \gamma M_1^{-1}x_3 + M_1^{-1}b \;;\; M_2^{-1}N_1x_1 \;;\; \gamma^{-1}|x_1 + x_2|\right),$$

where $\mathbf{x} = (x_1; x_2; x_3)$ with $x_i \in \mathbb{R}^n$ for $i = 1, 2, 3$. It is not difficult to verify that the mapping "dist$(\cdot,\cdot)$" defined by

$$\text{dist}(\mathbf{x}, \mathbf{y}) := \sum_{i=1}^{3} \|x_i - y_i\|,$$

is a metric[3] on $\mathbb{R}^{3n}$ for given $\mathbf{x} = (x_1; x_2; x_3)$ and $\mathbf{y} = (y_1; y_2; y_3)$, where $x_i, y_i \in \mathbb{R}^n$ $(i = 1, 2, 3)$. For $(x_1; x_2; x_3) \in \mathbb{S}$, straightforward algebraic computations reveal that

[3]The mapping "dist" can be regarded as an extension of taxicab or Manhattan distance from L^1 norm to L^2 norm for block vectors.

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$$\begin{aligned}\|M_1^{-1}N_2x_2+\gamma M_1^{-1}x_3+M_1^{-1}b\|+\|M_2^{-1}N_1x_1\|+\|\gamma^{-1}|x_1+x_2|\| &\leq (\beta+\gamma^{-1})\|x_1\|+(\alpha+\gamma^{-1})\|x_2\| \\ &\quad +\delta\gamma\|x_3\|+\delta\|b\| \\ &\leq \kappa(\|x_1\|+\|x_2\|+\|x_3\|)+\delta\|b\| \\ &\leq \frac{\kappa\delta\,\|b\|}{1-\kappa}+\delta\|b\| \\ &= \frac{\delta\,\|b\|}{1-\kappa}.\end{aligned}$$

As a result, one can deduce that $\mathcal{T}(x_1;x_2;x_3)$ maps $\mathbb{S}$ into $\mathbb{S}$.

It is worth mentioning that a fixed point of the mapping $\mathcal{T}$ is a solution of (2.5). Evidently, the set $\mathbb{S}$ is a compact, which ensures that ($\mathbb{S}$, dist) is a complete metric space. Hence, to conclude the assertion from the well-known Banach fixed-point theorem, we further show that $\mathcal{T}$ is a contracting mapping with respect to the metric dist$(\cdot,\cdot)$. Indeed, one need to verify that for some $\eta\in[0,1)$, the following inequality holds:

$$\text{dist}\,(\mathcal{T}(\mathbf{x}),\mathcal{T}(\mathbf{y}))\leq\eta\ \text{dist}(\mathbf{x},\mathbf{y}), \tag{2.12}$$

for $\mathbf{x}=(x_1;x_2;x_3)$ and $\mathbf{y}=(y_1;y_2;y_3)$ in $\mathbb{S}$. It is immediate to see that

$$\begin{aligned}\text{dist}\,(\mathcal{T}(\mathbf{x}),\mathcal{T}(\mathbf{y})) &= \|M_1^{-1}N_2(x_2-y_2)+\gamma M_1^{-1}(x_3-y_3)\|+\|M_2^{-1}N_1(x_1-y_1)\| \\ &\quad +\gamma^{-1}\||x_1+x_2|-|y_1+y_2|\| \\ &\leq \|M_1^{-1}N_2(x_2-y_2)+\gamma M_1^{-1}(x_3-y_3)\|+\|M_2^{-1}N_1(x_1-y_1)\| \\ &\quad +\gamma^{-1}\|(x_1-y_1)+(x_2-y_2)\| \\ &\leq \|M_1^{-1}N_2(x_2-y_2)+\gamma M_1^{-1}(x_3-y_3)\|+\|M_2^{-1}N_1(x_1-y_1)\| \\ &\quad +\gamma^{-1}\|x_1-y_1\|+\gamma^{-1}\|x_2-y_2\| \\ &\leq (\beta+\gamma^{-1})\|x_1-y_1\|+(\alpha+\gamma^{-1})\|x_2-y_2\|+\delta\gamma\|x_3-y_3\| \\ &\leq \kappa\left(\|x_1-y_1\|+\|x_2-y_2\|+\|x_3-y_3\|\right),\end{aligned}$$

which shows that the inequality (2.12) satisfies for $\eta=\kappa$ where κ is given by (2.8). Thus, the desired result follows immediately. ∎

For the sake of *theoretical* derivations, we summarize Eqs. (2.5) in the succeeding matrix form

$$\mathcal{A}\mathbf{x}=\begin{bmatrix}M_1 & -N_2 & -\gamma I\\ -N_1 & M_2 & 0\\ D(\bar{x}+\bar{y}) & D(\bar{x}+\bar{y}) & -\gamma I\end{bmatrix}\begin{bmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{bmatrix}=\begin{bmatrix}b\\ 0\\ 0\end{bmatrix}=\mathbf{b}. \tag{2.13}$$

For the coefficient matrix $\mathcal{A}$ in (2.13), we consider the block triangular splitting $\mathcal{A}=\mathcal{M}-\mathcal{N}$ as follows:

$$\mathcal{M}=\begin{bmatrix}M_1 & 0 & 0\\ -N_1 & M_2 & 0\\ \tau D(\bar{x}+\bar{y}) & \tau D(\bar{x}+\bar{y}) & -\gamma I\end{bmatrix}\quad\text{and}\quad\mathcal{N}=\begin{bmatrix}0 & N_2 & \gamma I\\ 0 & 0 & 0\\ (\tau-1)D(\bar{x}+\bar{y}) & (\tau-1)D(\bar{x}+\bar{y}) & 0\end{bmatrix},$$

where τ is a real parameter. This special splitting leads to the following new iterative method to solve (1.1):

$$\begin{cases}\bar{x}^{(k+1)}=M_1^{-1}(N_2\bar{y}^{(k)}+\gamma\bar{z}^{(k)}+b)\\ \bar{y}^{(k+1)}=M_2^{-1}N_1\bar{x}^{(k+1)}\\ \gamma\bar{z}^{(k+1)}=-\left((\tau-1)|\bar{x}^{(k)}+\bar{y}^{(k)}|-\tau|\bar{x}^{(k+1)}+\bar{y}^{(k+1)}|\right),\end{cases} \tag{2.14}$$

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for $k = 0, 1, 2, \ldots$, assuming that $(\bar{x}^{(0)}; \bar{y}^{(0)}; \bar{z}^{(0)})$ is given such that $\bar{z}^{(0)} = (1/\gamma)|\bar{x}^{(0)} + \bar{y}^{(0)}|$. Each step of (2.14) requires the exact solution of the linear system of equations with the coefficient matrices M_1 and M_2. Here, we choose matrices M_1 and M_2, which are relatively easy to invert and apply in comparison with A. It should be noted that the iterative method (2.14) is not a practical option when splitting $A = M_1(N_1 = 0)$.

The following theorem establishes sufficient conditions for the convergence of iterative scheme (2.14), as well as for the existence of a solution to (1.1). More precisely, if iteration (2.14) converges to $(\ell_1; \ell_2; \ell_3)$, then $\ell_1 + \ell_2$ is a solution of (1.1) by Proposition 2.1. Therefore, the theorem also provides a sufficient condition for the solvability of (1.1). In the proof of theorem, we need the following lemma.

LEMMA 2.3. [23] *Consider the quadratic equation $x^2 - bx + c = 0$, where b and c are real numbers. Both roots of the equation are less than one in modulus if and only if $|c| < 1$ and $|b| < 1 + c$.*

THEOREM 2.4. *Let α, β, and δ be defined as in (2.9) such that $(1+\beta)\delta + \alpha\beta < 1$. If the parameter τ belongs to the interval (ϕ_1, ϕ_2) with*

$$\phi_1 := \frac{(\alpha\beta - 1) + (1+\beta)\delta}{2(1+\beta)\delta} \quad \text{and} \quad \phi_2 := \frac{(1-\alpha\beta) + (1+\beta)\delta}{2(1+\beta)\delta}, \tag{2.15}$$

then the iterative method (2.14) converges for any initial guess and its limit provides a solution to (1.1).

Proof. Notice that the assumption $(1+\beta)\delta+\alpha\beta < 1$ ensures that $\phi_2 > 1$ and $\phi_1 < 0$. Let $(\bar{x}^{(k)}; \bar{y}^{(k)}; \bar{z}^{(k)})$ and $(\bar{x}^{(k-1)}; \bar{y}^{(k-1)}; \bar{z}^{(k-1)})$ be two successively generated triplets by the iterative method (2.14). We consider block vector $(e_1^{(k)}; e_2^{(k)}; e_3^{(k)})$ as follows:

$$e_1^{(k)} = \bar{x}^{(k)} - \bar{x}^{(k-1)}, \quad e_2^{(k)} = \bar{y}^{(k)} - \bar{y}^{(k-1)}, \quad e_3^{(k)} = \bar{z}^{(k)} - \bar{z}^{(k-1)}.$$

Evidently, we obtain the following three relations

$$\begin{aligned}
e_1^{(k+1)} &= M_1^{-1}N_2e_2^{(k)} + \gamma M_1^{-1}e_3^{(k)},\\
e_2^{(k+1)} &= M_2^{-1}N_1e_1^{(k+1)},\\
\gamma e_3^{(k+1)} &= -\Big((\tau-1)\Big(|\bar{x}^{(k)} + \bar{y}^{(k)}| - |\bar{x}^{(k-1)} + \bar{y}^{(k-1)}|\Big) - \tau\Big(|\bar{x}^{(k)} + \bar{y}^{(k)}| - |\bar{x}^{(k+1)} + \bar{y}^{(k+1)}|\Big)\Big),
\end{aligned}$$

which result in

$$\|e_1^{(k+1)}\| \le \alpha\|e_2^{(k)}\| + \gamma\delta\|e_3^{(k)}\|, \tag{2.16a}$$

$$\|e_2^{(k+1)}\| \le \alpha\beta\|e_2^{(k)}\| + \beta\gamma\delta\|e_3^{(k)}\|, \tag{2.16b}$$

$$\|e_3^{(k+1)}\| \le \frac{1}{\gamma}|\tau-1|\,\|e_1^{(k)}\| + \frac{1}{\gamma}\Big(|\tau-1| + |\tau|\alpha(1+\beta)\Big)\|e_2^{(k)}\| + |\tau|\delta(1+\beta)\|e_3^{(k)}\|, \tag{2.16c}$$

where α, β, and γ are given in (2.9). The relations (2.16a)–(2.16c) can be written in the condensed form

$$\begin{bmatrix} \|e_1^{(k+1)}\| \\ \|e_2^{(k+1)}\| \\ \|e_3^{(k+1)}\| \end{bmatrix} \le \underbrace{\begin{bmatrix} 0 & \alpha & \gamma\delta \\ 0 & \alpha\beta & \beta\gamma\delta \\ \frac{1}{\gamma}|\tau-1| & \frac{1}{\gamma}\Big(|\tau-1| + |\tau|\alpha(1+\beta)\Big) & |\tau|\delta(1+\beta) \end{bmatrix}}_{\mathcal{G}} \begin{bmatrix} \|e_1^{(k)}\| \\ \|e_2^{(k)}\| \\ \|e_3^{(k)}\| \end{bmatrix}. \tag{2.17}$$

For the notional simplicity, we set $\mathcal{E}_k = (\|e_1^{(k)}\|; \|e_2^{(k)}\|; \|e_3^{(k)}\|)$. Bearing in mind that $\mathcal{G}$ is a nonnegative matrix, the above inequality ensures that

$$\mathcal{E}_{k+1} \le \mathcal{G}^k\mathcal{E}_1.$$

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To conclude the assertion, we show that $\{(\bar{x}^{(k)};\bar{y}^{(k)};\bar{z}^{(k)})\}_{k=1}^{\infty}$ is a Cauchy sequence. To do so, we need to show that $\lim_{k\to\infty}\|\mathcal{E}_k\| = 0$.

Notice that $\lim_{k\to\infty}\mathcal{G}^k = 0$ if and only if $\rho(\mathcal{G}) < 1$. Hence, we need to show that $|\lambda| < 1$ for any arbitrary nonzero eigenvalue λ of $\mathcal{G}$. To determine the eigenvalues of $\mathcal{G}$, we use the Laplace expansion to write its characteristic polynomial as

$$f(\lambda) = \det(\lambda I - \mathcal{G}) = \lambda\Big(\lambda^2 - \big((1+\beta)\delta|\tau| + \alpha\beta\big)\lambda + (-1-\beta)\delta|\tau-1|\Big). \tag{2.18}$$

The nonzero eigenvalues are the root of the quadratic equation

$$\lambda^2 - \big((1+\beta)\delta|\tau| + \alpha\beta\big)\lambda + (-1-\beta)\delta|\tau-1| = 0. \tag{2.19}$$

From (2.19), we have

$$\lambda_1 = \frac{(1+\beta)\delta|\tau| + \alpha\beta - \sqrt{\Delta}}{2} \quad \text{and} \quad \lambda_2 = \frac{(1+\beta)\delta|\tau| + \alpha\beta + \sqrt{\Delta}}{2},$$

where $\Delta = \big((1+\beta)\delta|\tau| + \alpha\beta\big)^2 + 4(1+\beta)\delta|\tau-1|$. We observe that

$$\lambda_1 \to 0 \quad \text{and} \quad \lambda_2 \to (1+\beta)\delta + \alpha\beta,$$

as $\tau \to 1$. According to the assumption on τ, we readily obtain

$$1 - \frac{1}{(1+\beta)\delta} < \frac{(\alpha\beta-1)+(1+\beta)\delta}{2(1+\beta)\delta} \quad \text{and} \quad \frac{(1-\alpha\beta)+(1+\beta)\delta}{2(1+\beta)\delta} < 1 + \frac{1}{(1+\beta)\delta},$$

which imply that

$$|\tau - 1| < \frac{1}{(1+\beta)\delta}.$$

Using Lemma 2.3, we need to verify that

$$(1+\beta)\delta|\tau| + \alpha\beta < 1 - (1+\beta)\delta|\tau-1|.$$

Indeed, we need to show that the following inequalities hold

$$(1+\beta)\delta\tau + \alpha\beta < 1 - (1+\beta)\delta(\tau-1) \quad \text{for} \quad \tau > 1, \tag{2.20}$$

$$(1+\beta)\delta\tau + \alpha\beta < 1 - (1+\beta)\delta(1-\tau) \quad \text{for} \quad 0 \le \tau \le 1, \tag{2.21}$$

$$-(1+\beta)\delta\tau + \alpha\beta < 1 - (1+\beta)\delta(1-\tau) \quad \text{for} \quad \tau < 0. \tag{2.22}$$

In light of the assumption $\tau \in (\phi_1, \phi_2)$ where ϕ_1 and ϕ_2 are given by (2.15), it becomes immediately clear that the relations (2.20) and (2.22) are fulfilled. Also, the relation (2.21) can be readily derived from the fact that $(1+\beta)\delta + \alpha\beta < 1$. Now, we can deduce that $\mathcal{G}^k$ goes to zero as $k \to \infty$. This ensures that $\{(\bar{x}^{(k)};\bar{y}^{(k)};\bar{z}^{(k)})\}_{k=1}^{\infty}$ is convergent. Now, let $(\ell_1;\ell_2;\ell_3)$ denote the limit of sequence $\{(\bar{x}^{(k)};\bar{y}^{(k)};\bar{z}^{(k)})\}_{k=1}^{\infty}$ constructed by (2.14). One can verify that $(\ell_1;\ell_2;\ell_3)$ satisfies (2.5). Hence, $\ell_1 + \ell_2$ is a solution of (1.1) by Proposition 2.1, and we can conclude the assertion. ∎

Under certain conditions, Theorem 2.2 shows that Eq. (1.1) has a unique solution within the set $\mathbb{S}$. Theorem 2.4 provides sufficient conditions under which iterative method (2.14) converges to a solution of

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(1.1). Consequently, when the AVE has a unique solution and the assumptions of Theorems 2.2 and 2.4 are met, the proposed iterative method is guaranteed to converge to the solution of Eq. (1.1) that lies within $\mathbb{S}$.

In [12, Proposition 4], it is shown that AVE (1.1) is uniquely solvable for any b when $\|A^{-1}\| < 1$. In the numerical experiments, the AVEs in Examples 3.1–3.3 have unique solution, which can be verified by the following proposition [15, Proposition 2.1].

PROPOSITION 2.5. *Let A be nonsingular and $A = M - N$ be a splitting of A such that $\|M^{-1}\| + \|M^{-1}N\| < 1$. Then, $\|A^{-1}\| < 1$.*

REMARK 2.6. *Notice that asymptotic convergence rate of (2.14) does not rely on γ. Indeed, by Eq. (2.18) in the proof of Theorem 2.4, it is seen that the value of γ does not affect the convergence rate of the iterative method (2.14). This theoretical conclusion is also observed in our numerical experiments.*

Although Theorem 2.4 establishes sufficient criteria for convergence of the proposed method, numerical results show that it remains effective well beyond these conditions. In particular, for problems having multiple solutions, our numerical observations illustrate that the method converges under a wider set of circumstances. The remark below presents some alternative, though less easily verifiable, sufficient conditions for the convergence. We leave complete theoretical analysis of these extended cases for future study.

REMARK 2.7. *Let the system (2.5) is solvable and $(\bar{x}^*; \bar{y}^*; \bar{z}^*)$ be an arbitrary solution of it. Let us apply the iterative method (2.14) with $\bar{y}^{(0)} = M_2^{-1}N_1\bar{x}^{(0)}$. For $k = 1, 2, \ldots$, it follows that:*

$$\bar{x}^{(k+1)} = M_1^{-1}N_2M_2^{-1}N_1\bar{x}^{(k)} + \gamma M_1^{-1}\bar{z}^{(k)} + M_1^{-1}b, \tag{2.23}$$

and

$$\gamma\bar{z}^{(k)} = (1-\tau)|(I + M_2^{-1}N_1)\bar{x}^{(k-1)}| + \tau|(I + M_2^{-1}N_1)\bar{x}^{(k)}|.$$

Substituting the above relation into equation (2.23), we obtain

$$\bar{x}^{(k+1)} = \bar{T}\bar{x}^{(k)} + \tau M_1^{-1}|(I + M_2^{-1}N_1)\bar{x}^{(k)}| + (1-\tau)M_1^{-1}|(I + M_2^{-1}N_1)\bar{x}^{(k-1)}| + M_1^{-1}b, \tag{2.24}$$

with $\bar{T} = M_1^{-1}N_2M_2^{-1}N_1$. Thus, it can be deduced that iteration scheme (2.14) is convergent iff the above iterative method converges. To analyze the convergence properties of (2.24), we define $\bar{e}^{(k)} := \bar{x}^{(k)} - \bar{x}^*$. We assume that $M_2 \neq -N_1$, which implies that $I + M_2^{-1}N_1$ is nonzero. Letting $1 - \frac{1}{2\varepsilon} < \tau < 1$ where $\varepsilon := \|M_1^{-1}\|\|\mathcal{W}\|$ with $\mathcal{W} := (I + M_2^{-1}N_1)$, it can be verified that

$$\|\bar{e}^{(k+1)}\| \leq \|\mathcal{F}(\bar{x}^{(k)}) - \mathcal{F}(\bar{x}^*)\| + (1-\tau)\varepsilon\|\bar{e}^{(k-1)}\|, \tag{2.25}$$

where

$$\mathcal{F}(x) = \bar{T}x + \tau M_1^{-1}|(I + M_2^{-1}N_1)x|.$$

Consider the case that $\varepsilon < 1$ which holds in the numerical examples presented in Section 3. Assume that there exists a neighborhood $\mathbb{D}_{\bar{\delta}}$ of $\bar{x}^*$, defined by $\mathbb{D}_{\bar{\delta}} = \{x \mid \|x - \bar{x}^*\| < \bar{\delta}\}$, such that $\mathcal{F}(x)$ is Lipschitz continuous in $\mathbb{D}_{\bar{\delta}}$ with constant $\bar{L} := 1 - 2(1-\tau)\varepsilon$. That is,

$$\|\mathcal{F}(x) - \mathcal{F}(\bar{x}^*)\| \leq \bar{L}\|x - \bar{x}^*\|,$$

for all $x \in \mathbb{D}_{\bar{\delta}}$. In view of (2.25) and the above relation, it is observed that

$$\|\bar{e}^{(k+1)}\| \leq \bar{L}\|\bar{e}^{(k)}\| + (1-\tau)\epsilon\|\bar{e}^{(k-1)}\|. \tag{2.26}$$

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TABLE 1
Splitting matrices for Examples 3.1 and 3.2

M_1	N_1	M_2	N_2
diag(A)	$M_1 - A$	tridiag(A)	$M_2 - A$

Notice that if $x^{(j)} \in \mathbb{D}_{\bar{\delta}}$, *for* $j = k-1, k$, *then Eq.* (2.25) *together with the preceding relation imply that* $\|\bar{e}^{(k+1)}\| < \bar{\delta}$, *which ensures that* $x^{(k+1)} \in \mathbb{D}_{\bar{\delta}}$. *Consequently, as long as* $x^{(0)}, x^{(1)} \in \mathbb{D}_{\bar{\delta}}$ *the entire sequence of approximate solutions* $\{x^{(\ell)}\}_{\ell=1}^{\infty}$ *produced by* (2.14) *remains within* $\mathbb{D}_{\bar{\delta}}$. *By the relation* (2.26), *we deduce that*

$$\begin{bmatrix} \|\bar{e}^{(k+1)}\| \\ \|\bar{e}^{(k)}\| \end{bmatrix} \leq \bar{\mathcal{G}} \begin{bmatrix} \|\bar{e}^{(k)}\| \\ \|\bar{e}^{(k-1)}\| \end{bmatrix}, \quad k = 1, 2, \ldots, \tag{2.27}$$

where

$$\bar{\mathcal{G}} := \begin{bmatrix} \bar{L} & (1-\tau)\epsilon \\ 1 & 0 \end{bmatrix}.$$

The characteristic polynomial of $\bar{\mathcal{G}}$ *is given by*

$$P_{\bar{\mathcal{G}}}(\lambda) = \lambda^2 - \bar{L}\lambda - (1-\tau)\epsilon.$$

Since $(1-\tau)\varepsilon < 1$ *and* $0 < \bar{L} < 1 - (1-\tau)\varepsilon$, *Lemma 2.3 ensures that* $\rho(\bar{\mathcal{G}}) < 1$. *Thus, from Eq.* (2.27), *we deduce that* $\lim_{k\to\infty} \|\bar{e}^{(k)}\| = 0$.

3. Numerical experiments. In this section, we present comparison results to numerically illustrate the feasibility of proposed method and its superiority over iterative schemes in [3, 19]. The numerical experiments have been carried out on an Intel Core i7-1355U CPU @ 1.70 GHz processor and 16.0GB RAM using MATLAB.R2023b.

In the following, the MATLAB notation "tril(A,-1)" stands for the strictly lower triangular matrix whose nonzero entries are the elements of A below the main diagonal. By "tridiag(A)," we mean a tridiagonal matrix, which is constructed by keeping only the three central diagonals of A in place and setting all other entries to zero.

In the numerical experiments of Examples 3.1 and 3.2, the proposed method is implemented using the splittings provided in Table 1. We comment that applying the act of inverse of M_1 (M_2) is cheaper than the matrix A.

In following tables, the required number of iterations and the elapsed CPU time in seconds are, respectively, reported under "IT" and "CPU". The initial guess is taken to be the zero vector in the implementations of all fixed-point methods. Let $x^{(k)}$ denote the kth approximate solution computed by either iterative method (1.2) or (1.3), the corresponding iterations are stopped once

$$\frac{\|b + |x^{(k)}| - Ax^{(k)}\|}{\|b\|} < 10^{-8},$$

or the maximum number of iteration steps exceeds 1000. The iterations of proposed method (2.14) is terminated as soon as

$$\frac{\|b + |\bar{x}^{(k)} + \bar{y}^{(k)}| - A(\bar{x}^{(k)} + \bar{y}^{(k)})\|}{\|b\|} < 10^{-8},$$

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or maximum allowed of 1000 iteration number reached, where $(\bar{x}^{(k)}; \bar{y}^{(k)}; \bar{z}^{(k)})$ stands for kth triplet approximation computed by (2.14). In view of Remark 2.6, we report the numerical experiments for the case $\gamma = 5$.

By Proposition 2.5 and considering the splitting $A = M_1 - N_1$, it can be verified for the test problems in Examples 3.1–3.3 that $\|A^{-1}\| < 1$, which ensures the existence of a unique solution to the AVE. The condition $(1+\beta)\delta + \alpha\beta < 1$ in Theorem 2.4 was numerically verified for small problem sizes in Examples 3.1 and 3.2. For all numerical tests, the condition $\|M_1^{-1}\|\|I + M_2^{-1}N_1\| < 1$ from Remark 2.7 is satisfied.

Let x^* denote the exact solution of the AVE. For Examples 3.1–3.3, which each has a unique solution, we further report the relative error of the computed approximate solution under "Err" where

$$\text{Err} := \frac{\|x^* - x^{(k)}\|}{\|x^*\|},$$

in which $x^{(k)}$ is the kth approximate solution computed by (1.2) or (1.3). When implementing the iteration scheme (2.14), we set

$$\text{Err} := \frac{\|x^* - (\bar{x}^{(k)} + \bar{y}^{(k)})\|}{\|x^*\|},$$

where $\bar{x}^{(k)}$ and $\bar{y}^{(k)}$ are computed at the kth step of (2.14).

The MATLAB backslash is used to solve all the linear system of equations. For all iterative methods in Examples 3.1 and 3.2, the optimal value of the parameter τ (denoted as τ_{es} in the tables) was determined experimentally for the smallest problem sizes, and its value was not changed when the sizes grow.

The right-hand side vector b in (1.1) is generated such that $b = Ax^* - |x^*|$ where x^* are, respectively, set to be $x^* = (1, 2, \ldots, n)^T$ and $x^* = (-1, 1, -1, \ldots, 1)^T$ in Examples 3.1 and 3.2.

EXAMPLE 3.1. [18, 19] *We consider the AVE* (1.1) *such that the matrix* $A \in \mathbb{R}^{n\times n}$ $(n = m^2)$ *is given by*

$$A = T_x \otimes I_m + I_m \otimes T_y + pI_n,$$

where $T_x = \text{tridiag}\{-1-r, 4, -1+r\}$, $T_y = \text{tridiag}\{-1-r, 0, -1+r\}$, $r = \frac{q}{2(m+1)}$ *and* $m > 0$ *is prescribed.*

Tables 2 and 3 display the results obtained from applying iteration schemes (1.2), (1.3) and proposed method for solving (1.1) associated with Example 3.1. As seen, the proposed method outperforms the iterative approaches developed in [3, 19].

In [3], the convergence of the iterative method (1.3) is analyzed under the assumption that $\tau \in (0, 2)$. Nevertheless, as previously commented, the value of τ that yields the fastest convergence in practice may fall outside $(0, 2)$. To be more specific, as shown in Tables 2 and 3, the experimentally optimal values of τ identified in Example 3.1 exceed two. For the sake of completeness, Table 4 also includes results for cases where $\tau \in (0, 2)$. As seen, the parameter τ_{es} can significantly improve the convergence speed of (1.3) compared to restricting the assumption $\tau \in (0, 2)$.

The following example is taken from [6, 7, 10], the test problem is reduced to the previous example when $\mu = p$ and $q = 0$.

EXAMPLE 3.2. *Let* $m > 0$ *be a prescribed integer,* $n = m^2$ *and* $\mu > 0$ *is a given positive constant. We consider the absolute value equations* (1.1) *such that* $A = \hat{A} + \mu I$ *where* $\hat{A} = \text{tridiag}(-I, S, -I) \in \mathbb{R}^{n\times n}$ *with* $S = \text{tridiag}(-1, 4, -1) \in \mathbb{R}^{m\times m}$.

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TABLE 2
Numerical results for Example 3.1 with $(q,p)=(1,2)$ *and* $Q=-0.9I$

Approach	τ_{es}		m = 32	64	128	256
Iterative method (1.3)	2.99	IT	13	14	14	14
		CPU	0.0277	0.1366	1.0908	4.4834
		Err	9.0289e-09	3.1914e-09	3.7250e-09	4.0009e-09
Iterative method (1.2)	1	IT	8	8	8	8
		CPU	0.0175	0.0834	0.6200	2.5521
		Err	3.3454e-09	3.9888e-09	4.3233e-09	4.4933e-09
Proposed method (Iterative method (2.14))	1.6	IT	23	23	23	23
		CPU	**0.0091**	**0.0324**	**0.1221**	**0.5013**
		Err	8.4625e-09	5.9866e-09	4.4002e-09	3.4068e-09

TABLE 3
Numerical results for Example 3.1 with $(q,p)=(1,4)$ *and* $Q=-0.9I$

Approach	τ_{es}		m = 32	64	128	256
Iterative method (1.3)	2.5	IT	7	7	7	7
		CPU	0.0145	0.0917	0.4063	3.7286
		Err	3.7627e-09	3.4868e-09	3.5424e-09	3.6188e-09
Iterative method (1.2)	1.1	IT	7	7	7	7
		CPU	0.0147	0.0835	0.3966	3.9095
		Err	1.1245e-09	1.1635e-09	1.1830e-09	1.1927e-09
Proposed method (Iterative method (2.14))	1.22	IT	13	13	13	13
		CPU	**0.0048**	**0.0196**	**0.0812**	**0.2681**
		Err	1.9744e-09	2.7389e-09	3.3526e-09	3.6916e-09

TABLE 4
Numerical results for DRS method (Iteration (1.3)) when τ *belongs to established convergence interval in [3] for Example 3.1*

τ	(q,p)		m = 32	64	128	256
1.99	(1,2)	IT	26	27	27	27
		CPU	0.0594	0.2931	2.0132	10.690
		Err	1.0460e-08	6.7986e-09	7.6485e-09	8.0835e-09
1.99	(1,4)	IT	14	14	14	14
		CPU	0.0238	0.1311	0.7372	5.7132
		Err	3.7256e-09	4.1512e-09	4.3686e-09	4.4783e-09

The numerical results for Example 3.2 are summarized in Tables 5 and 6. Overall, with suitable splittings $A=M_1-N_1=M_2-N_2$, the proposed method is superior to the iterative schemes examined in [3, 19] for solving (1.1), as demonstrated in Tables 2–6. In particular, Fig. 1 shows the performance of the proposed method as a function of the parameter $\tau\in[0.1,2]$ for Example 3.1. The reported results in Tables 2 and 3 together with the CPU times in Fig. 1 show that the proposed method requires less CPU time for all tested values of $\tau\in[0.1,2]$ than the iterative methods in [3, 19] using their optimal parameter values. It is worth noting that each iterate of the proposed method is computationally cheaper than an iteration of the methods in [3, 19].

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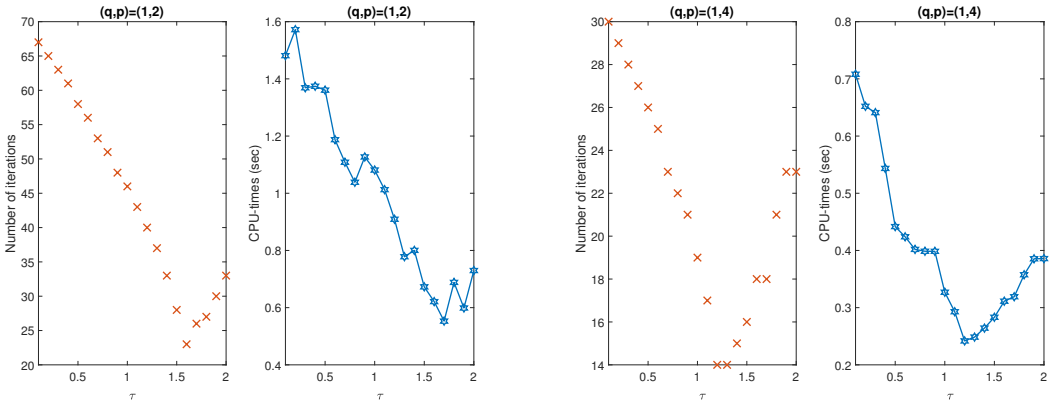

FIGURE 1. *Required number of iterations and CPU times of the proposed method versus τ in Example 3.1 ($m = 256$).*

TABLE 5
Numerical results for Example 3.2 with $\mu = 4$ and $Q = -0.9I$

			m			
Approach	τ_{es}		32	64	128	256
Iterative method (1.3)	1.99	IT	11	11	11	11
		CPU	0.0072	0.0285	0.3043	2.2878
		Err	6.8707e-09	7.2023e-09	7.3677e-09	7.4503e-09
Iterative method (1.2)	0.89	IT	12	12	12	12
		CPU	0.0080	0.0361	0.3432	2.5448
		Err	3.3495e-09	3.1080e-09	3.0453e-09	3.0297e-09
Proposed method (Iterative method (2.14))	1.1	IT	12	12	12	12
		CPU	**0.0036**	**0.0160**	**0.0724**	**0.1964**
		Err	4.5791e-09	5.1826e-09	5.4854e-09	5.6371e-09

TABLE 6
Numerical results for Example 3.2 with $\mu = 2$ and $Q = 0.9I$

			m			
Approach	τ_{es}		32	64	128	256
Iterative method (1.3)	1.99	IT	15	15	15	15
		CPU	0.0088	0.0488	0.5107	3.7751
		Err	1.1272e-08	1.2387e-08	1.2944e-08	1.3223e-08
Iterative method (1.2)	1.2	IT	14	14	14	14
		CPU	0.0087	0.0513	0.5044	4.1147
		Err	2.8134e-08	2.4570e-08	2.2418e-08	2.1219e-08
Proposed method (Iterative method (2.14))	1.2	IT	18	18	18	18
		CPU	**0.0054**	**0.0291**	**0.0699**	**0.2332**
		Err	2.4146e-08	2.9200e-08	3.1714e-08	3.2969e-08

We observe that the iteration steps of the examined methods remain constant, indicating all approaches exhibit size-independent convergence behavior for solving (1.1) in Examples 3.1 and 3.2. From Eq. (2.19), the eigenvalues of $\mathcal{G}$ rely on α, β, and δ for a prescribed value of τ. In Examples 3.1 and 3.2, we observe that α, β, and δ do not change as the problem size grows. This allowed us to determine near-optimal parameters

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TABLE 7
Numerical results for Example 3.3 with $Q = I$

Approach	τ		n = 50	500	1000	2000
Iterative method (1.3)	1.99	IT	5	5	5	5
		CPU	0.0006	0.0355	0.1167	0.6343
		Err	7.8653e-10	1.1027e-09	1.7965e-09	4.5909e-09
Iterative method (1.2)	1	IT	5	5	5	6
		CPU	0.0006	0.0352	0.1172	0.9473
		Err	3.3861e-09	6.4325e-09	1.3285e-08	1.9842e-09
Proposed method (Case I) (Iterative method (2.14))	0.9	IT	6	8	10	16
		CPU	0.0004	0.0138	0.0338	0.2250
		Err	1.9360e-09	4.1093e-09	6.4409e-09	5.9595e-09
Proposed method (Case II) (Iterative method (2.14))	0.9	IT	6	7	9	13
		CPU	**0.0002**	**0.0122**	**0.0306**	**0.2091**
		Err	1.7741e-09	7.1639e-09	5.5818e-09	1.3611e-08

of iterative methods using small problem (grid) sizes and then exploit the same parameters to larger sizes. For general unstructured matrices, however, parameter selection appears more problem-dependent. Thus, for the following two test problems, we use a fixed default values, leaving more thorough parameter analysis as future work.

To evaluate the robustness of the proposed methods in more general settings, we report numerical results for AVE with a randomly generated coefficient matrix. To ensure the reproducibility of the results, all random matrices are generated using a fixed random seed (rng(0) in MATLAB).

EXAMPLE 3.3. [9, 21] *We construct a random matrix A according to the following* MATLAB *command,*

$$A = \text{round}(100\,(\text{eye}(n) - 0.02\,(2\,\text{rand}(n) - 1)))\,.$$

The right-hand side b is constructed as follows $b = Ax^* - |x^*|$ where $x^* \in \mathbb{R}^n$ is a random vector.

In Example 3.3, the proposed method is applied to solve the AVE with $M_1 = 100I$. The choice of M_2 is examined in the following two cases:

- Case I. $M_2 = \text{tridiag}(A)$
- Case II. $M_2 = 100I + \text{tril}(A, -1)$.

The numerical results, for solving the AVE in Example 3.3, are reported in Table 7. As seen, in both Cases I and II, the proposed method outperforms the iterative methods developed in [3, 19] with respect to required CPU time for convergence. For further details, Fig. 2 displays the required number of iterations for convergence with respect to the parameter τ. For this particular problem, the iteration count remains largely insensitive to the choice of τ within the interval $[-2, 2]$.

In the previous examples, the solution of each AVE was unique. In Example 3.5, we numerically examine the performance of the proposed method for a test problem with multiple solutions. To construct this test problem, we employ the following proposition, established in [12, Proposition 6].

PROPOSITION 3.4. *If $b < 0$ and $\|A\|_\infty < \hat{\gamma}/2$ where $\hat{\gamma} = \min_i |b_i| / \max_i |b_i|$, then AVE has exactly 2^n distinct solutions, each of which has no zero components and a different sign pattern.*

EXAMPLE 3.5. *Let $n = 1000$. We consider the AVE problem (1.1) such that $A = [a_{ij}]$ is an $n \times n$ matrix such that*

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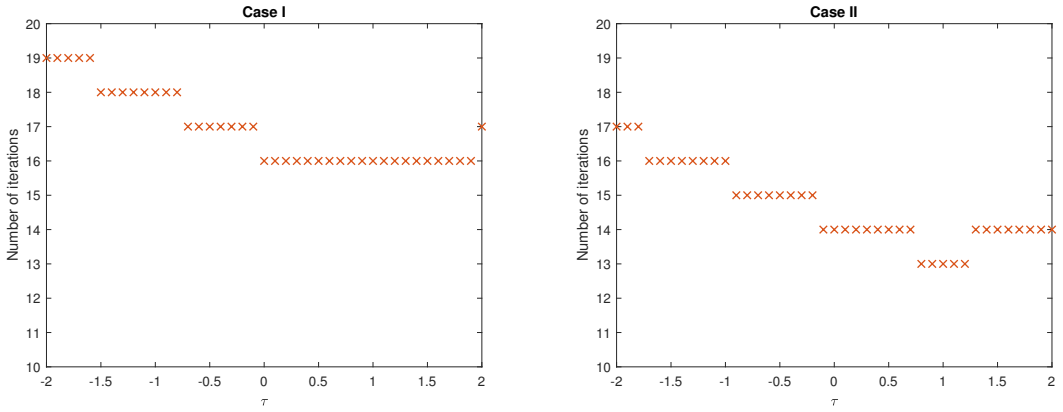

FIGURE 2. *Required number of iterations of the proposed method versus τ in Example 3.3 for $n = 2000$.*

TABLE 8
Numerical results for the proposed method (Iteration (2.14)) for Example 3.5

τ		D_1	D_2	D_3	D_4
0.9	IT	16	18	18	18
	CPU	0.0579	0.0698	0.0821	0.0717
	Res	2.4996e-09	1.9103e-09	5.0732e-10	5.1099e-10

$$a_{ij} = 0.5 \cdot 10^{-4} \cdot \left(1 + \frac{i+j}{n}\right),$$

and $b = -\text{ones}(n, 1)$.

For the matrix A in Example 3.5, one can see that $\|A\|_\infty < 0.5$, which ensures the existence of 2^n solutions of the corresponding AVE. Let D_A denote the diagonal matrix composed of the diagonal elements of A. We apply the proposed method with $M_1 = D_i(D_A + 5I)$ and $M_2 = D_i(D_A + 2I + \text{tril}(A, -1))$ for $i = 1, 2, 3, 4$ in which $D_1 = I$, $D_2 = -I$, and the diagonal matrices D_3 and D_4 are defined such that

$$D_3(\ell, \ell) = (-1)^\ell \quad \text{and} \quad D_4(\ell, \ell) = (-1)^{\ell+1}, \qquad \ell = 1, 2, \ldots, n.$$

Our numerical experiments show that iterative method (1.3) fails to converge in Example 3.5. The iterative method (1.2) converges after 11 iterations (with a CPU time of 0.2599 seconds) to an approximate solution $\hat{x}^*$ of (1.1) in which all entries are negative. The corresponding residual norm is $\|b - A\hat{x}^* + |\hat{x}^*|\| = 3.3846e{-}10$. It should be noted that the auxiliary matrix D_i ensures the convergence of iteration (2.14) to the exact solution x_i, where $\text{sign}(x_i(\ell)) = -D_i(\ell, \ell)$ for $\ell = 1, 2, \ldots, n$.

Table 8 reports the numerical results of applying the proposed method for determining four different solutions of (1.1). The symbol "Res" in the table is defined by

$$\text{Res} := \|b + |\bar{x}^{(k)} + \bar{y}^{(k)}| - A(\bar{x}^{(k)} + \bar{y}^{(k)})\|.$$

As observed, the proposed method can be effectively applied to solve the AVE in Example 3.5, and it outperforms the iterative method (1.2) in terms of the computational time needed for convergence.

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4. Concluding remarks. We introduced a new class of iterative techniques for solving absolute value problems of the form $Ax - |x| = b$, which are implicitly associated with applying a fixed-point method for solving a three-by-three nonlinear block system of equations. The proposed approach is inspired by a Kellogg-type iterative method, recently developed in [20] for linear systems of the form $Ax = b$. We proved that the proposed iterative methods converge to the solution of AVE under certain conditions. In addition, it was numerically illustrated that upon suitable choice of the splitting matrices, the new approach needs relatively smaller CPU time for convergence comparing with the two recently examined methods in [3] and [19].

In Remark 2.2, we provided some sufficient conditions for convergence. This analysis extends beyond problems with unique solutions, providing an early foundation for understanding convergence behavior. Nevertheless, a comprehensive and rigorous theoretical analysis of convergence under broader, nonunique cases remains an important direction for future work.

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