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ERRATUM FOR THE PAPER "SOLUTION OF SYMMETRIC POSITIVE SEMIDEFINITE PROCRUSTES PROBLEM" [ELECTRON. J. LINEAR ALGEBRA 35 (2019) 543–554]*

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Abstract. We give an example showing that the main result, i.e., Theorem 2.12, in the paper by Peng, Wang, Peng, and Chen [Electron. J. Linear Algebra 35 (2019) 543–554] is not correct. We also indicate where the mistake in the proof occurs.

1. Introduction. Denote by S^n_+ the cone of $n \times n$ positive semidefinite matrices and by $\|\cdot\|$ the Frobenius norm. Given $F, G \in \mathbb{R}^{n\times n}$, the positive semidefinite procrustes problem asks for $P \in S^n_+$ that minimizes $\|F - PG\|$, if such a minimizer exists.

For simplicity, we will only consider the case where F is symmetric and G is diagonal and of full rank. Then, as shown in [1], a minimizer of $\|F - PG\|$ among the real symmetric $n \times n$ matrices is given by

$$\hat{S} = \Phi \odot (FG + GF),$$

where Φ is the matrix with

$$\Phi_{i,j} = \frac{1}{G^2_{i,i} + G^2_{j,j}},$$

and where $\odot$ denotes the Hadamard product.

To find a positive semidefinite minimizer P of $\|F - PG\|$, the following method is proposed in [2, Theorem 2.12]: Compute the eigendecomposition

$$\hat{S} = N\Gamma N^{\mathsf{T}},$$

and set

$$P = N\Gamma_+ N^{\mathsf{T}},$$

where $(\Gamma_+)_{i,i} = \max\{0, \Gamma_{i,i}\}$. In other words, to find a positive semidefinite minimizer, compute a symmetric minimizer and project it onto the cone of positive semidefinite matrices.

In Section 2, we give an example showing that this does not always give a minimizer and in Section 3 we point out where the mistake in the proof of [2, Theorem 2.12] occurs.

2. Counterexample. Consider

$$F = \begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix}, \quad G = \begin{pmatrix} 1 & 0 \\ 0 & 100 \end{pmatrix}.$$

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Then,

$$\Phi = \begin{pmatrix} 1/2 & 1/10001 \\ 1/10001 & 1/20000 \end{pmatrix},$$

and

$$\hat{S} = \begin{pmatrix} -1 & 404/10001 \\ 404/10001 & -1/100 \end{pmatrix}.$$

The determinant of the symmetric matrix $\hat{S}$ is positive and its trace is negative, so $\hat{S}$ is negative definite, which means $P = 0$.

Consider the positive semidefinite matrix

$$\tilde{P} = \begin{pmatrix} 4/10 & 4/100 \\ 4/100 & 4/1000 \end{pmatrix}.$$

Then

$$\|F - \tilde{P}G\|^2 = \frac{12251}{625} < 34 = \|F\|^2,$$

showing that $P = 0$ is not a minimizer of $\|F - PG\|$ among the positive semidefinite matrices.

3. Mistake in the proof. Below [2, Equation (2.20)], it is claimed that for positive semidefinite matrices $\bar{F}_1$ and $\bar{F}_2$ with $\mathrm{tr}(\bar{F}_1\bar{F}_2) = 0$ and a positive definite diagonal matrix D, the positive semidefinite minimizer X of

$$\|(\bar{F}_1 - X)D\|^2 + 2\mathrm{tr}(D\bar{F}_2XD),$$

is given by $X = \bar{F}_1$. If $\mathrm{tr}(D\bar{F}_2XD)$ would be nonnegative for all positive semidefinite X, the above claim would be true, but this is not the case. The claim is only true for the special case where all diagonal elements of matrix D and thus all nonzero singular values σ_i of matrix G are equal.

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