ERRATUM TO "PERIODIC TWO-DIMENSIONAL DESCRIPTOR SYSTEMS"*

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Abstract. We point out a mistake in a corollary of the above paper.

Key words. Multi-dimensional systems, periodic Schur form, commuting systems, simultaneous triangularization.

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The matrix $\Phi := D^{-1}CB^{-1}A$ of the periodic system

$$Bx_{k+1,\ell} = Ax_{k,\ell}, \quad Dx_{k+1,\ell+1} = Cx_{k+1,\ell}$$

is called its *monodromy matrix*. The following corollary, stated in [1] as Corollary 4.3, is only true for the "if" part. The "only if" part is not correct.

COROLLARY 4.3. *Let the $n \times n$ complex matrices B and D be invertible and let the monodromy matrix $\Phi := D^{-1}CB^{-1}A$ be a simple matrix. Then there exists a periodic Schur form*

$$\begin{bmatrix} -\hat{A} & z\hat{B} \\ z\hat{D} & -\hat{C} \end{bmatrix} := \begin{bmatrix} Z_1 & \\ & Z_2 \end{bmatrix} \begin{bmatrix} -A & zB \\ zD & -C \end{bmatrix} \begin{bmatrix} Q_1^* & \\ & Q_2^* \end{bmatrix}, \tag{4.1}$$

where $Q_1 = Q_2$ if and only if the matrices $B^{-1}A$ and $D^{-1}C$ commute.

That the condition $Q_1 = Q_2$ is not sufficient for the commutativity of $B^{-1}A$ and $D^{-1}C$, follows from the following counter example. If the matrices A, B, C and D are all upper triangular and B and D are invertible, then the pencil (4.1) is already in its periodic Schur form and $Q_1 = Q_2 = Z_1 = Z_2 = I_n$. But this would imply that all such upper-triangular matrices commute, which is obviously incorrect.

The references to this corollary in [1] should also be replaced by references to Theorem 4.1, which proves the "if" part. These corrections do not affect the other results of this paper.

REFERENCES

[1] P. Benner and P. Van Dooren. Periodic Two-Dimensional Descriptor Systems. *Electronic Journal of Linear Algebra*, **39**:472–490, 2023.

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